

- 1) The equation of the tangent line to the graph of $y = x + e^x$ at $x = 0$ is
a) $y = x$ b) $y = x + 1$ c) $y = x + 2$ d) $y = 2x$ e) $y = 2x + 1$
- 2) $\int_{-1}^0 \sqrt{1-x^2} dx$ is
a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\sqrt{2}}{2}$ d) $\sqrt{2}$ e) none of these
- 3) $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2+1}}{3x-2}$ is
a) -1 b) $-\frac{1}{3}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$ e) 1
- 4) A particle's acceleration for $t \geq 0$ is given by $a(t) = 12t + 4$. The particle's initial position is 2 and its velocity at $t = 1$ is 5. The position of the particle at $t = 2$ is
a) 4 b) 10 c) 12 d) 16 e) 20
- 5) $\int_{-3}^3 |x+1| dx$ is
a) 0 b) 5 c) 10 d) 15 e) 20
- 6) The linear approximation to the curve $f(x) = 1 + 3x \ln\left(\frac{x}{2}\right)$ at $x = 2$ is
a) $y = 3x - 5$ b) $y = 3x + 1$ c) $y = 6x - 11$
d) $y = 6x + 5$ e) none of these

- 7) The area bounded by the curve $y = x^3 + 2x^2 - 4x - 8$ and the x -axis is
- a) $\frac{28}{3}$ b) $\frac{40}{3}$ c) $\frac{52}{3}$ d) $\frac{64}{3}$ e) none of these
- 8) The greatest possible area of a triangular region with one vertex at the center of a circle of radius 1 and the other two vertices on the circle is
- a) $\frac{1}{2}$ b) 1 c) $\sqrt{2}$ d) π e) $\frac{1+\sqrt{2}}{4}$
- 9) Let f be a function defined by $f(x) = \int_0^{x^2} e^{x+t} dt$ for all real numbers x .
Then $f'(1) =$
- a) $e-1$ b) e^2 c) $e^2 - e$ d) $2e^2$ e) $3e^2 - e$
- 10) The second derivative of the function $f(x) = 2\sin x + 3\cos^2 x$ at $x = \pi$ is
- a) -7 b) -6 c) -4 d) 3 e) 6
- 11) $\int_0^1 \frac{x}{x^2+1} dx$ is
- a) 1 b) $\frac{\pi}{4}$ c) $\tan^{-1} \frac{\sqrt{2}}{2}$ d) $\ln 2$ e) $\ln \sqrt{2}$
- 12) The value of $\int_{-\pi}^{\pi} x^3 \sin(x^2) dx$ is
- a) -1 b) 0 c) 1 d) does not exist e) none of these
- 13) The slope of the tangent line to the graph of $4x^2 + cx + 2e^y = 2$ at $x = 0$ is 4.
The value of c is
- a) -8 b) -4 c) 2 d) 4 e) 8

14) The average value of the function $f(x) = (2x + 3)^2$ on the interval $[-3, -1]$ is

- a) -4 b) $\frac{4}{3}$ c) $\frac{7}{3}$ d) $\frac{14}{3}$ e) 5

15) If $f(x) = \int_e^x \ln(t) dt$ for all positive x , then $f'(x) =$

- a) x b) $\frac{1}{x}$ c) $\ln x$ d) $x \ln x$ e) $x \ln x - 1$

16) The area of the region bounded by the coordinate axes and the tangent line to the graph of $y = \frac{1}{8}x^2 + \frac{1}{2}x + 1$ at the point $(0, 1)$ is

- a) $\frac{1}{16}$ b) $\frac{1}{8}$ c) $\frac{1}{4}$ d) 1 e) 2

17) $\int_1^6 \frac{3x}{\sqrt{x+3}} dx$ is

- a) 5 b) 10 c) 15 d) 20 e) 25

18) The base of an isosceles triangle with equal sides of 4 cm is increasing at the rate of $\frac{1}{8}$ cm per second. The rate at which the vertex angle is changing (in radians per second) at the instant the base is also 4 cm is

- a) $\frac{\sqrt{3}}{16}$ b) $\frac{\sqrt{3}}{24}$ c) $\frac{\sqrt{3}}{32}$ d) $\frac{\sqrt{3}}{40}$ e) $\frac{\sqrt{3}}{48}$

19) Let f be a function such that $f(x) = f(1-x)$ for all real numbers x . If f is differentiable everywhere, then $f'(0) =$

- a) $f(0)$ b) $f(1)$ c) $-f(0)$ d) $f'(1)$ e) $-f'(1)$

20) The curve $x^2 - 2xy + 4y^2 = 3$ has a vertical tangent line at

- a) $x = -4$ b) $x = -2$ c) $x = -1$ d) $x = 0$ e) none of these

21) $\lim_{x \rightarrow 0} \frac{\tan x - x}{x^3}$ is

- a) $\frac{1}{6}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) 1 e) none of these

22) The area of the region bounded by the curves $y = 2\sqrt{x+3}$ and $y = |x|$ is

- a) $\frac{32}{3}$ b) $\frac{44}{3}$ c) $\frac{56}{3}$ d) $\frac{68}{3}$ e) none of these

23) The integral with the greatest value on the interval $\left[0, \frac{\pi}{4}\right]$ is

- a) $\int_0^{\frac{\pi}{4}} \sin x \, dx$ b) $\int_0^{\frac{\pi}{4}} \cos x \, dx$ c) $\int_0^{\frac{\pi}{4}} \cos^2 x \, dx$
d) $\int_0^{\frac{\pi}{4}} \cos 2x \, dx$ e) $\int_0^{\frac{\pi}{4}} \sin x \cos x \, dx$

24) Let $f(x) = \begin{cases} -x^2 + 4x - 2, & \text{if } x < 1 \\ -x^2 + 2, & \text{if } x \geq 1 \end{cases}$.

The statement which is true about f is

- a) f has an absolute maximum at $x = 0$.
b) f has an absolute maximum at $x = 1$.
c) f has an absolute maximum at $x = 2$.
d) f has no absolute maximum.
e) f has a local maximum at both $x = 0$ and $x = 2$.

25) If f is a continuous function on the set of real numbers and if a and b are real numbers, then the statement(s) which are true is

I. $\int_a^b f(x) dx = \int_{a+3}^{b+3} f(x-3) dx$

II. $\int_a^b f(x) dx = \int_a^3 f(x) dx - \int_b^3 f(x) dx$

III. $\int_{3a}^{3b} f(x) dx = 3 \int_a^b f(3x) dx$

a) I only b) II only c) I and II only d) II and III only e) I, II, III

26) The value of $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos^2 x}$ is

a) 0 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) 1 e) none of these

27) If $f(x) = \frac{x}{x + \frac{c}{x}}$, then $f'(x)$ is

a) $-\frac{2cx}{(x^2 + c)^2}$ b) $\frac{2cx}{(x^2 + c)^2}$ c) $\frac{2c}{(x^2 + c)^2}$

d) 1 e) none of these

28) The value of $\int_4^9 \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx$ is

a) $\frac{44}{3}$ b) $\frac{85}{2} + \ln \frac{9}{4}$ c) $\frac{4}{3}$ d) 2 e) none of these

29) A particle moves along the x-axis so that its position at time t in seconds, is given by $x(t) = t^2 - 7t + 6$. The velocity of the particle is zero for t equal to

a) 1 b) 6 c) 1 or 6 d) 3.5 e) 1 or 3.5 or 6

- 30) The coordinates of the point of inflection on the graph of $y = x^3 - 15x^2 + 33x + 100$ are
- a) (9, 0) b) (5, -48) c) (1, 119) d) (9, -89) e) (5, 15)
- 31) The value of c that satisfies the Mean Value Theorem for Derivatives on the interval $[0, 5]$ for $f(x) = x^3 - 6x$ is
- a) $-\frac{5}{\sqrt{3}}$ b) 0 c) 1 d) $\frac{5}{3}$ e) $\frac{5}{\sqrt{3}}$
- 32) The value of $\lim_{x \rightarrow -4^-} \frac{|x + 4|}{x + 4}$ is
- a) -1 b) 0 c) 1 d) 8 e) none of these
- 33) If $f(x) = \ln(\cos(3x))$, then $f'(x)$ is
- a) $-3 \csc(3x)$ b) $3 \sec(3x)$ c) $3 \tan(3x)$
- d) $-3 \tan(3x)$ e) $-3 \cot(3x)$
- 34) The value of $\int_0^{2\pi} |\sin x| dx$ is
- a) 4 b) 2 c) 1 d) 0 e) none of these
- 35) If $x [f(x)]^3 + x f(x) = 6$ and $f(3) = 1$, then $f'(3)$ is
- a) $-\frac{1}{6}$ b) $-\frac{11}{3}$ c) $\frac{1}{2}$ d) 1 e) none of these
- 36) If $g(t) = (2 - 3t)^{-\frac{1}{2}}$, then $g''(0)$ is
- a) $\frac{3}{32}$ b) $\frac{27}{32}$ c) $\frac{27}{16\sqrt{2}}$ d) $\frac{3}{16\sqrt{2}}$ e) none of these

- 37) $\lim_{x \rightarrow 0} (1-2x)^{\frac{1}{x}}$ is
- a) -2 b) 1 c) e d) e^{-2} e) none of these
- 38) The value of $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$ is
- a) 6 b) 2 c) e d) 4 e) none of these
- 39) The function f , given by $f(x) = x^4 + 4x^3$, is decreasing on the interval
- a) $(-3, 0)$ b) $(0, \infty)$ c) $(-3, \infty)$ d) $(-\infty, -3)$
e) $(-\infty, 0)$
- 40) If the function $f(x) = \begin{cases} 3ax^2 + 2bx + 1 & \text{if } x \leq 1 \\ ax^4 - 4bx^2 - 3x & \text{if } x > 1 \end{cases}$ is differentiable for all real values, then b is equal to
- a) $-\frac{11}{4}$ b) $\frac{1}{4}$ c) $-\frac{7}{16}$ d) 0 e) $-\frac{1}{4}$

